similar to that used by Punga. Using the Nomenclature of Punga, the following velocities can be written:

$$\left.\begin{aligned} \frac{d\mathbf{R}^{(G)}}{dt} &= \frac{d\mathbf{R}^{(0)}}{dt} + \omega \times \mathbf{r}^{(G)} + \frac{\delta \mathbf{r}^{(G)}}{\delta t} \\ \frac{d\mathbf{R}_N}{dt} &= \frac{d\mathbf{R}^{(0)}}{dt} + \omega \times \mathbf{r}_N + \frac{\delta \mathbf{r}_N}{\delta t} \\ \mathbf{v}_e &= \frac{d\mathbf{R}^{(0)}}{dt} + \omega \times \mathbf{r}_N + \mathbf{v}_e^{(0)} \end{aligned}\right\} \quad (7)$$

where $\mathbf{v}_e^{(0)}$ is the velocity of the escaping gases relative to the reference systems fixed in the body. Using these relations, Eq. (6) will reduce to

$$\bar{F} = M\frac{d\mathbf{V}^*}{dt} - \frac{d^2M}{dt^2}(\mathbf{r}_N - \mathbf{r}^{(G)}) - \frac{dM}{dt}\left\{\mathbf{v}_e^{(0)} + \frac{\delta \mathbf{r}_N}{\delta t} - 2\frac{\delta \mathbf{r}^{(G)}}{\delta t} - 2\,\omega \times (\mathbf{r}_N - \mathbf{r}^{(G)})\right\} \quad (8)$$

The last two terms of Eq. (8) are usually labeled as the reactive force since they are functions of mass ejection. Now if we call the reactive force **K**, then Eq. (8) reduces to

$$\mathbf{F} + \mathbf{K} = M(d\mathbf{V}^*/dt) \quad (9)$$

which is the usual form of the rocket equation. Examining Eq. (8), the motion of the center of mass cannot be separated from the reactive force.

There are a few printing errors that crept into Punga's paper, which do not affect the final equation derived.

References

[1] Punga, V., "Motion of the center of gravity of a variable-mass body," AIAA J. **2**, 1482 (1964).
[2] Thorpe, J. F., "On the momentum theorem for a continuous system of variable mass," Am. J. Phys. **30**, 637–640 (1962).
[3] Rankin, R. A., "The mathematical theory of the motion of rotated and unrotated rockets," Phil. Trans. Roy. Soc. London, **A241**, 457–585 (March 1949).
[4] Leitmann, G., "On the equation of rocket motion," J. Brit. Interplanet. Soc. **16**, 141–147 (1957).
[5] Haftman, R. L., *Dynamics* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1962), Vol. 1, 1st ed., pp. 153–160.

Spectroscopic Constants for the N^+ Ion

M. S. Wecker*
General Applied Science Laboratories, Inc., Westbury, N. Y.

THE purpose of this note is to point out an error in Ref. 1. Stupochenko, et al., in their Table 1, list incorrect spectroscopic data for the ion N^+. The authors cite Ref. 2 as the source of their data, and their error stems from the fact that they transcribed Moore's data for N^{++} (NIII) rather than that for N^+ (NII).

The data in Table 1, from Ref. 2, should be substituted for that in Ref. 1. As an example of the magnitude of error involved in using Ref. 1 rather than Ref. 2, we compute the partition function of N^+ at several temperatures (see Table 2).

$$Q_{N+} = \sum_i g_i\, e^{-\epsilon_i/kT}$$

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* Senior Scientist. Member AIAA.

Table 1 Energy levels for N^+ ion

ϵ_p(cm^{-1})	g_p
0	1
49.1	3
131.3	5
15315.7	5
32687.1	1
47167.7	5
92237.9	7
92251.3	5
92252.9	3

Table 2 N^+ partition function

	Q_{N^+}	
T(°K)	Ref. 1	Ref. 2
1,000	5.11	7.94
6,000	5.84	8.93
12,000	5.92	9.72

References

[1] Stupochenko, E. V., Stakhanov, I. P., Samuilov, E. V., Pleshanov, A. S., and Rozhdestvenskii, I. B., "Thermodynamic properties of air in the temperature interval from 1000 to 12,000°K and the pressure intervals from 0.001 to 1000 atm.," ARS J. **30**, 98–112 (1960); also Predvoditelev, A. S. (ed.), *Physical Gas Dynamics* (Pergamon Press, New York, 1961), pp. 1–40.
[2] Moore, C. E., "Atomic energy levels," Nat. Bur. Std. (U.S.) Circ. 467, 32–44 (June 1949).

Viscous Flow Properties on Slender Cones

R. A. Peter,* M. D. High†, and E. F. Blick‡
University of Oklahoma, Norman, Okla.

Nomenclature

M = Mach number
P = static pressure
Re = Reynolds number
T = temperature
U = velocity
β = $(M_1^2 - 1)^{0.5}$
δ^* = boundary-layer displacement thickness
γ = ratio of specific heats
θ = semivertex angle
σ = Prandtl number

Subscripts

1 = freestream conditions
2 = condition at outer edge of boundary layer
c = inviscid conical flow value
w = wall
x = distance from vertex along cone surface

THE use of a similarity parameter has been employed by several authors to predict the inviscid flow properties for supersonic and hypersonic flow over cones. Linnell and

Received July 30, 1964; revision received November 18, 1964.
* Graduate Student, School of Aerospace and Mechanical Engineering. Member AIAA.
† Instructor; now Research Engineer, ARO Inc., Tullahoma, Tenn. Member AIAA.
‡ Assistant Professor, School of Aerospace and Mechanical Engineering. Member AIAA.

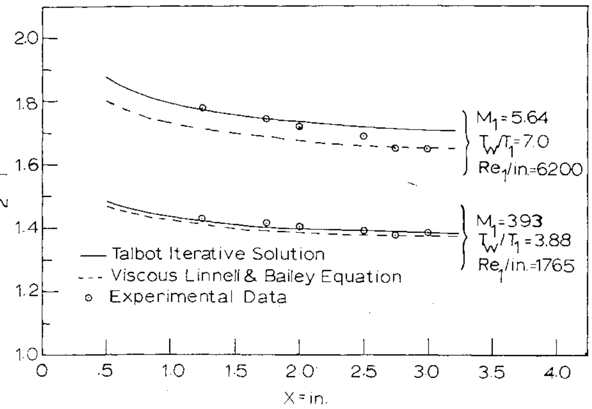

Fig. 1 Pressure ratio on a 5° half-angle cone.

Bailey[1] used the similarity concept to obtain the following expression for the pressure ratio at zero angle of attack,

$$P_c/P_1 = 1 + (\gamma M_1^2/2)(4\sin^2\theta_c)(2.5 + 8\beta\sin\theta_c)/(1 + 16\beta\sin\theta_c) \quad (1)$$

Blick[2] has given expressions for temperature, velocity, and Mach number for cones using the parameter $M_1 \sin\theta_c$. They are,

$$T_c/T_1 = [1 + 0.35\,(M_1\sin\theta_c)^{1.5}] \text{ for } M_1\sin\theta_c \leq 1.0 \quad (2a)$$

and, for $M_1 \sin\theta_c \geq 1.0$,

$$T_c/T_1 = [1 + \exp(-1 - 1.52\,M_1\sin\theta_c)][1 + (M_1\sin\theta_c)^2/4] \quad (2b)$$

$$U_c/U_1 = \cos\theta_c\,[1 - \sin\theta_c/M_1]^{0.5} \quad (3)$$

$$M_c/M_1 = (U_c/U_1)(T_1/T_c)^{0.5} \quad (4)$$

To calculate the induced pressure on a cone due to the thickening of the boundary layer, one could use the method of Talbot[3] which involves an iteration on the flow variables at the edge of the boundary layer. Peter,[4] utilizing Eqs. (1-4) in a computer program based on Talbot's method, was able to obtain expressions for the boundary-layer displacement thickness gradient $d\delta^*/dx$ and the induced pressure on cones at zero angle of attack. His results are plotted in Fig. 1 vs the experimental data of King.[5]

By using Eqs. (1–4) and Talbot's[3] equation for boundary-layer displacement thickness,

$$(\delta^*/x) = (3.012/3^{1/2})(C/Re_2)^{1/3}\{(\pi/2)[T_W/T_2 - [\sigma(\gamma - 1)/4]M_2^2] - [1.0 + \sigma^{1/3}(T_W/T_2 - T_{AW}/T_2)]\} \quad (5)$$

it is possible to obtain a semiempirical equation for $d\delta^*/dx$ based on freestream properties and wall temperatures only. [Note that Eq. (5) is a function of properties at the edge of the boundary layer and hence are unknown at the start of an iterative solution.] This direct equation is

$$d\delta^*/dx = [0.342(T_W/T_1)^{1.3} + 0.08M_1^2][3.9M_1^{1/2}(Re_{1x})^{0.27} \times \{\theta_c^2 M_1 (Re_{1x})^{1/2} + 0.342(T_W/T_1)^{1.3} + 0.08M_1^2\}^{1/2}]^{-1} \quad (6)$$

Eq. (6) includes the transverse curvature effect of Hill[6] and is valid for a Prandtl number of 0.725.

The viscous solution for flow properties over slender cones can be obtained quickly by simply substituting θ_2 for θ_c in the inviscid equations [Eqs. (1–4)], where

$$\theta_2 = \theta_c + \tan^{-1}(d\delta^*/dx) \quad (7)$$

and where $d\delta^*/dx$ is obtained from Eq. (6). This means that the useful Linnell-Bailey equation [Eq. (1)] can be applied in either the viscous or inviscous regime.

Studies made over a Mach number range of 3–20, a Reynolds number range of 10^3–10^6, and wall temperature range of 1–9 showed that this simplified method gave pressure errors of less than 6% when compared with the lengthy iterative solutions.

A further comparison was made with the experimental data of King.[5] The results shown in Fig. 1 indicate that the simplified method (labeled the viscous Linnell and Bailey curve) falls within 3% of the experimental data.

References

[1] Linnell, R. D. and Bailey, J. Z., "Similarity-rule estimation methods for cones and parabolic noses," J. Aeronaut. Sci. **23,** 796–797 (1956).

[2] Blick, E. F., "Similarity rule estimation method for cones," AIAA J. **1,** 2415 (1963).

[3] Talbot, L., Koga, T., and Sherman, P. M., "Hypersonic viscous flow over slender cones," NACA TN 4327 (September 1958).

[4] Peter, R. A., "Pressure distribution on a cone at zero angle of attack with mass injection," Master's Thesis, University of Oklahoma, Norman, Okla. (1964).

[5] King, H. H., "Hypersonic flow over a slender cone with gas injection," TR HE-150-205, University of California, Berkeley, Calif. (1962).

[6] Hill, J. A. F., "Mach number measurements in high speed wind tunnels," AGARDograph 22 (October 1956).

Reply to A. P. Cappelli

Robert E. Lavender*

NASA George C. Marshall Space Flight Center, Huntsville, Ala.

Nomenclature

D = landing gear diameter
k = radius of gyration about center of gravity
k_e = effective spring constant per leg parallel to the vehicle's longitudinal axis
L_1 = overturning radius ($0.5D$ cos45° for 2-2 impact)
L_2 = original height of center of gravity
LF = deceleration load factor based on earth gravity for landing on level surface with all legs crushing simultaneously
N = number of legs
V_v = vertical velocity
V_h = horizontal velocity
W_e = earth weight of vehicle
θ = lunar slope, negative for downhill landing
φ, $\dot\varphi$ = vehicle attitude and attitude rate, positive nose up
μ = coefficient of friction

Cappelli[1] questions the discontinuity in the stability profile shown by the writer[2] since he has never observed any such discontinuity. There are basically two reasons for this. First, the stability profile is usually determined with a degree of accuracy (or rather lack of accuracy) as demonstrated by Cappelli in his Fig. 1.[1] There could very easily be a discontinuity in his profile of Fig. 1 at a vertical velocity of 8 fps, for example, without discovering it, since no analytical results were obtained at this velocity. In fact, even if re-

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* Scientific Assistant to Director, Aero-Astrodynamics Laboratory. Associate Fellow Member AIAA.